Derivation of the approximate relationship between GDP, Productivity, and Labor Force changes and change in the rate of Unemployment.

In the following let:

Y = GDP	$
P = Productivity	$/Person-hour
U = Unemployment	%
F = Labor Force	Persons
H = Average Labor Hours	hours

Δ prefix denotes the absolute difference in the variable's value between consecutive time periods.

Start with a standard identity:

(1) $Y = P(1-U)FH$

Then assuming $\Delta H = 0$
We can approximate the product rule for differentiation with

(2) $\Delta Y \approx \Delta P(1-U)FH - \Delta UPFH + \Delta FP(1-U)H$

Dividing (2) by (1) to get a percentage change from the previous period:

(3) $\Delta Y/Y \approx (\Delta P(1-U)FH - \Delta UPFH + \Delta FP(1-U)H) / P(1-U)FH$

Dividing out the terms on the right side by the denominator:

(4) $\Delta Y/Y \approx \Delta P/P - \Delta U/(1-U) + \Delta F/F$

Rearranging:

(5) $\Delta U/(1-U) \approx \Delta P/P + \Delta F/F - \Delta Y/Y$

or

(6) $\Delta U \approx (1-U) (\Delta P/P + \Delta F/F - \Delta Y/Y)$

Since 1-U is typically close to 1 you can approximate this by:

(7) $\Delta U \approx \Delta P/P + \Delta F/F - \Delta Y/Y$

If the desire is that $\Delta U = 0$ then it must be (approximately) the case that:

(8) $\Delta Y/Y = \Delta P/P + \Delta F/F$

If the desire is to make ΔU negative then it must be (approximately) the case that:

(9) $\Delta Y/Y > \Delta P/P + \Delta F/F$

And if you want to change unemployment by X (i.e. a reduction would require that X be negative) then it must be (approximately) the case that:

(10) $\Delta Y/Y = \Delta P/P + \Delta F/F - X$